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***Abstract*—A new all-optical node architecture, known as *Packet Optical Add-Drop Multiplexer* (POADM), may lead to a considerable cost reduction for the infrastructure of the all-optical metropolitan rings if associated with proper dimensioning studies. We present a dimensioning problem which consists of minimizing the total number of receivers located in POADMs for a metropolitan all-optical ring with a fixed number of wavelengths and a given traffic matrix. We prove that this problem is NP-complete and provide a heuristic. The heuristic principle is to match and to group transmissions instead of considering them independently. We justify the transmission group matching approach by confronting the results of our algorithm with its simplified version. The results obtained allow us to recommend the heuristic in the planning of POADM configurations in all-optical rings with a limited number of wavelengths.**

I. Introduction

An all-optical network is composed of equipment which can handle the optical signal from its origin to its destination as an opposition to an opto-electronic network where the signal must be stopped and regenerated at each node. All-optical networks offer both better performance and lower energy consumption than the classical opto-electronic networks [4]. For these reasons, they have been chosen to be the next generation of metropolitan networks. The DOROthéE[1] project aims to reduce the *CAPital EXpenditure* (CAPEX) of metropolitan networks with low ecological impact by properly dimensioning them. We focus our attention on all-optical rings. In an all-optical network, there are two main parameters that have to be taken into consideration in the dimensioning process, namely the number of wavelengths and the equipment required in the nodes. Thus, this is a bi-criteria optimization problem. The well studied *Wavelength Assignment* (WA) problem considers the first parameter only. Numerous works [5], [8], [11] considered networks equipped with *Add-Drop Multiplexers* (ADMs). An ADM is both a receiver and a transmitter. The defined problems treated the number of wavelengths and the number of ADMs. Nevertheless in [6] the authors proposed a very promising node architecture, the *Packet Optical Add-Drop Multiplexer* (POADM), in which transmitters and receivers are independent. A node is composed of several receivers and one transmitter that can emit on each wavelength. Problems defined for ADM networks are no longer valid for POADM since the receivers and a transmitter are separated. The proper dimensioning of a POADM network requires the consideration of two parameters : the number of wavelengths and the number of receivers.

In our previous work [15], we provided a solution considering the number of receivers as a constraint. There, we defined a single criterium minimization problem known as the *Minimum WaveLength Problem* (MWLP). The problem was to find the minimum number of wavelengths needed for a given traffic matrix when the total number of receivers in the network is minimal. We showed that the problem is NP-complete and provided a heuristic. In the present work, we define the problem called the *Minimum Receiver Problem* (MRP) consisting in finding the minimum number of receivers needed for a given traffic matrix in a network where the total number of available wavelengths is fixed. Indeed, the maximum number of wavelengths that can be used is predetermined from the moment the fiber has been buried underground. As long as we do not use more than this number of wavelengths, the cost of using one more wavelength is negligible since the fiber is already present. In that case, the number of receivers becomes the only parameter, whereas the maximum number of wavelengths becomes an additional constraint. It should be noticed that there is a relation between these two problems. Indeed if a solution to the MWLP also respects the wavelength constraint (i.e. does not use more than a given number of wavelengths) then this solution is an optimal solution to the MRP. In the article we thus attempt to state formally and solve the MRP. We show that the problem is NP-complete and provides a heuristic.

The rest of this paper is organized as follows. Section II contains an overview of all-optical technology and a survey of the related research studies. In Section III we formally introduce the MRP. We study its complexity in Section IV and in Section V we present a heuristic algorithm which solves the MRP and comment on the results obtained in Section VI. Finally, we conclude and outline perspectives.

II. Related work

In this section we give an overview of all-optical technologies. In the first part, we explain the principle of traffic assignment and, in the second part, we present several node architectures which lead to different dimensioning problems. All but one of these architectures have been exhaustively studied.

[1]DOROthéE is a Digiteo project financed by the Ile-de-France region.

Optical networks use *Wavelength Division Multiplexing* (WDM) to carry a vast amount of traffic through the fiber. Each wavelength is considered as a high speed channel with a fixed transmission rate. Since the wavelength capacity is large, most of the dimensioning solutions consist in grouping requests in order to reduce the number of wavelengths required. These methods, known as *traffic grooming* methods, use *Time Division Multiplexing* (TDM) to divide wavelengths into smaller channels. The number of channels that a wavelength contains is called *grooming ratio*.

In point-to-point opto-electronic networks the traffic is add/drop to/from a wavelength using electronic *Add-Drop Multiplexers* (ADMs). The optical signal is systematically stopped at each node and thus an ADM is needed on each wavelength. Since the cost of these devices represents a large part of the *Capital Expenditure* (CAPEX), it is a priority to reduce the number of required ADMs. In the first technology we present, all optical network nodes are equipped with *Optical Add-Drop Multiplexers* (OADMs) that, added to a node, allow the optical signal to bypass the node. When equipped with OADMs, a node requires ADM only on the wavelengths on which it has to add/drop traffic. By properly organizing the traffic flow it is thus possible to suppress ADMs and then reduce the CAPEX. The traffic grooming and wavelength assignment problem was, for OADM node architecture, proved NP-complete [5] and heuristics were provided for mesh [13], ring [3], [5], [8], [11], [16], [10] and multi-ring [17] topologies. More recently, networks provided with *Reconfigurable Optical Add-Drop Multiplexers* (ROADMs) have been studied. A ROADM is a tunable device that can add/drop traffic onto/from different wavelengths over time. ROADM offers more flexibility than OADM and allows one to perform dynamic traffic grooming, saving both CAPEX and *Operational Expenditure* (OPEX). With dynamic traffic grooming, ROADM leads to new dimensioning constraints and thus to new dimensioning problems [7]. In [17] the authors considered an architecture provided with *Optical Cross Connect* (OXC) that allows time-slot traffic switching. Based on the observation that a node may have to add but not drop (or, symmetrically, drop but not add) traffic from a wavelength, the authors of [6] present another node architecture known as *Packet Optical Add-Drop Multiplexer* (POADM). This architecture, we study, is composed of several receivers and one tunable transmitter that can add traffic to different wavelengths over time. As for ADM based networks, the number of receivers used is a large part of the CAPEX. Once again, the flexibility provided by this technology should lead to better results if associated with effective traffic assignment heuristics. Nevertheless, there is no dimensioning process for this technology.

III. Minimum Receiver Problem

We formally introduce the MRP. Given a traffic matrix and a number of available wavelengths, the MRP is the minimization problem which consists in finding an assignment of the traffic on the wavelengths that minimizes the number of receivers required. The problem presented below is the decision problem associated with the MRP.

Problem: Minimum Receiver Problem

Data:

- An elementary circuit [2] $G = (V, E)$.
- A traffic matrix T where $T[i, j]$ is an amount of traffic to be sent from a node i to a node j.
- A set $\lambda = \{\lambda_1, \lambda_2, ..., \lambda_W\}$ of wavelengths, $W \in \mathbb{N}$.
- A wavelength capacity $C \in \mathbb{N}$.
- A number of receivers $z \in \mathbb{N}$.

We call *assignment* the operation which decomposes T into a set of W matrices T_k of the same dimension as T and associates T_k to a wavelength $k \in \lambda$ of a ring.

Question: Is it possible to find an assignment of the traffic T on the wavelengths λ that respects simultaneously the *flow* and *capacity* constraints and that does not use more than z receivers ?

Flow constraint: For any couple of nodes (i, j), the amount of traffic carried on each wavelength has to be equal to the total amount of traffic between i and j:

$$\forall i, j \in V, \sum_{k \in \lambda} T_k[i, j] = T[i, j].$$

Capacity constraint: Let $load_k(x)$ be the load of the arc $x \in E$ for the wavelength k. In other words $load_k(x)$ is equal to the amount of traffic carried by the arc x on the wavelength k. $load_k(x)$ cannot exceed the capacity C:

$$\forall k \in \lambda, \forall x \in E, \text{load}_k(x) = \sum_{i,j \ \ s.t. \ \ x \in path(i,j)} T_k[i, j] \leq C.$$

IV. Complexity

We use the *Partition Problem* (PP) [9] to prove that the MRP is NP-Complete.

Problem: Partition Problem

Data:

- A set of integers $X = \{x_1, x_2, ..., x_m\}$

Question: Is it possible to find A and B such that $A \cup B = X$, $A \cap B = \emptyset$ and $\sum_{x \in A} x = \sum_{x \in B} x$?

Theorem 4.1: The MRP is NP-complete.

Proof: Given an instance of MRP and an assignment of traffic T on λ, we can determine whether this assignment involves less than z receivers and verify both the capacity and flow constraints in polynomial time. The certificate of MRP is in P.

Let us consider an instance of the PP. From this instance we build an instance of the MRP as follows. For each integer x_i of X we create two vertices in an initially empty elementary circuit G. Those vertices are s_i (source of the traffic) and d_i (destination of the traffic). Nodes in G are ordered so that $s_1 \prec \ldots \prec s_m \prec d_1 \prec \ldots \prec d_m$ where $X \prec Y$ means that X is placed before Y in the ring. We build the traffic matrix so that the only non-zero elements are $T[s_i, d_i] = x_i$. We fix $W = 2$, $C = (\sum_{k=1}^{m} x_k)/2$ and $z = m$.

For the given instances, if there is a solution to the PP then we are able to build a solution to the MRP and reciprocally. We associate the subset A (respectively B) with the first λ_1 (respectively the second λ_2) wavelength. In the PP solution, an integer x_i is associated with one of the two subsets, A or B. The whole traffic $x_i = T[s_i, d_i]$ is therefore assigned to the wavelength which corresponds to this subset. As we assign the entire traffic flow to a single wavelength, the flow constraint is respected. For the same reason the solution does not use more than z receivers. For each wavelength the amount of traffic passing through an arc is less than or equal to the amount of traffic passing through the arc (s_m, d_1) on the same wavelength. Since $\text{load}_1(s_m, d_1) = \sum_{x \in A} = C$ and $\text{load}_2(s_m, d_1) = \sum_{x \in B} = C$, the capacity constraint is respected. If the PP has a solution then the MRP has a solution too.

If we have a solution to the MRP, then each traffic flow is assigned to a single wavelength due to the number of receivers limited to z. The assignment of traffic on the arc (s_m, d_1) directly provides the partition of integers, which is a solution to the PP. If there is a solution to the MRP, then there is also a solution to the PP.

We prove there is a polynomial reduction from the PP to the MRP and that the certificate of MRP is in P. Thus, we prove that MRP is NP-complete. ■

V. Heuristic

The algorithm we propose has three steps. First, it groups the requests, then selects groups and, finally, assigns them to wavelengths.

For a given n-node ring with nodes numbered from 1 to n, we consider each wavelength as an n-dimension cube. Dimension i is associated with the arc i (i.e. the arc between nodes i and $i{+}1$). The length of each edge of this cube is equal to C. Such a cube is called a *box* and the number of boxes available is equal to the number of wavelengths available, w. A request $r^{(x,y)}$ is a traffic flow between nodes x and y. It is represented as as an n-dimension vector. The height $h(r)$ of request r is equal to the amount of traffic this request is carrying and its length $l(r)$ is equal to the number of arcs between its origin and its destination. For example, in a 4-node ring we consider the request $r^{(1,3)} = (3, 3, 0, 0)$. This request passes through arcs 1 and 2 but not through arcs 3 and 4. Thus, we have $h(r^{(1,3)}) = 3$ and $l(r^{(1,3)}) = 2$. A *unitary* request is a request with height equal to one (i.e. the smallest amount of traffic to be transmitted). As non-unitary requests can be split over several wavelengths, we may consider without loss of generality that all requests are unitary requests.

A set of transmission requests with the same destination forms an *element*. Inside the element, requests are decreasingly ordered by length. An element e may be seen as a vector sum of the request vectors it contains. An element composed of all the requests towards a given destination is called a *complete* element. There are at most n complete elements numbered according to their destination node. The length of an element e is $l(e) = \max_{r \in e}(l(r))$ and its size is $s(e) = \sum_{r \in e} l(r)$. Its height $h(e)$ is equal to the number of unitary requests it contains. A *rectangular element* is a particular element that contains only requests of the same length.

In order to minimize the number of the receivers needed in the ring, the traffic associated with a given complete element should be carried by the smallest number of wavelengths. Our goal is therefore to cut the complete elements into elements that fit into the smallest number of available wavelengths. To discover the shape of a complete element e_d, we have to compute the amount of traffic passing through each arc towards d. Let us note l_i^d the amount of traffic in the arc i destined to d. Under the assumption that a node does not send anything to itself and taking into account a circular network architecture we obtains a vector l^d :

$$\begin{cases} l_i^d &= 0 & \text{if } i = d \\ l_i^d &= l_n^d + t_{i,d} & \text{otherwise and if } i = 1 \\ l_i^d &= l_{i-1}^d + t_{i,d} & \text{otherwise} \end{cases}$$

The three steps, that compose our algorithm, are repeated until the assignment of all traffic. The variable C_h represents the height of the cut.

Initialisation $C_h = C$.

Step 1 Generally speaking, heuristics of packing obtain better results when the elements to be packed have regular shapes. A *cut* provides a partition of an element e into a set of k *resultant* elements $\{e_0, e_1, \ldots, e_{k-1}\}$ with $k = \lceil h(e)/C_h \rceil$. Moreover, we want a cut to have some special properties in order to produce resultant elements with regular shape. Firstly, resultant elements should (as much as possible) be the same height. Ideally, this height is a sub-multiple of C. Secondly, resultant elements should be as low as possible in order to reduce the space they will take when packed into a box. Rectangular elements are, according to their definition, perfectly regular. We decide to measure how much an element is not "rectangular like". The measure of the *irregularity* of an element e is $\text{irr}(e) = h \cdot l(e) - s(e)$. The greater the irregularity number of an element is the harder it is to pack. The cut has to minimize $\sum_i \text{irr}(e_i) = \sum_i (h \cdot l(e_i) - s(e_i))$. The following method leads to a cut with these properties.

Informally, we form groups of C_h requests from the bottom to the top of e. Since the requests are ordered in e according to their lengths, resultant element e_i contains longer requests than a resultant element e_{i+1}. The element on the top can be smaller than h. Formally,

$$e_i = \begin{cases} \{r_{iC_h}, r_{iC_h+1}, \ldots, r_{iC_h+C_h-1}\} & \text{if } i \neq k-1 \\ \{r_{(k-1)C_h}, \ldots r_{h(e)}\} & \text{otherwise} \end{cases}$$

Step 2 only if $C_h > 1$**,** We use here an acceptance-rejection method to select groups of elements. The acceptance rate is noted τ. Ideally, we would like to consider each possible set (group) of elements. Nevertheless, as we want the complexity to remain reasonable, we consider hereafter only pairs of elements (or single element). If the elements of a pair do not fit together in a virtual empty box of capacity C_h then the pair is rejected regardless of τ. For elements a and b, from a non-rejected pair, we compute fit_rate$(a, b) = \frac{s(a)+s(b)}{nC_h}$ which measures the fraction of space occupied by the elements a and b when packed in a virtual box of capacity C_h. A pair of elements with a *fit rate* greater than τ is selected. As an element can appears into more than one pair, we use a maximum matching algorithm [12] (on the selected pairs) in order to get the biggest subset of accepted pairs that does not contain a same element twice. We notice that the elements of a same accepted pair are from this moment indivisible and will be treated as a single element. In the remaining subset of elements, an element a is accepted if fit_rate$(a) = \frac{s(a)}{nC_h} > \tau$.

Step 3 We use a first fit decreasing method [18] to pack all the accepted elements (or accepted pair of elements) into the w boxes. After the first iteration the boxes can already contain elements. The height of the cut is modified so that $h = \lfloor h/2 \rfloor$.

VI. RESULTS

In this section we discuss the performance of the heuristic algorithm. Firstly, we show, on instances, the influence of the acceptance rate τ. Afterwards we compare two variants of our heuristic: with and without pairing the elements in step 2 of the proposed algorithm. We show, thereby, the influence of the pairing of elements and explain in which cases it should be used. The experiments presented below have been made for 16-node ring networks with wavelength capacity $C = 32$. We use *All-To-All* (ATA) spatial traffic distribution, in which the sizes of the connections are generated following uniform or normal (N(μ,0.2μ), $\mu = 16$) distribution. In another simulation series we use *Rich-Get-Richer* (RGR) [1] spatial distribution for which the mean volume of traffic received by each node is equal to μ. The latter is chosen to represent a realistic traffic condition, as in a metropolitan ring some nodes may attract more traffic (e.g. video base server, backbone access node).

A. Influence of the acceptance rate

As we have said before, the number of receivers required for a given node is equal to the number of parts in which the associated complete element has to be cut. In order to minimize the number of receivers we want this number of parts to be as small as possible. In other words we want the resultant elements to be as high as possible. Nevertheless a too high resultant element may be difficult to pack if it does not fit well with other resultant elements. The acceptance rate τ allow us to select elements that, despite their height, do not lead to the degradation of the wavelength utilization. Figure 1 depicts the evolution of the ratio z/W (number of receivers / number of wavelengths) depending on the acceptance rate.

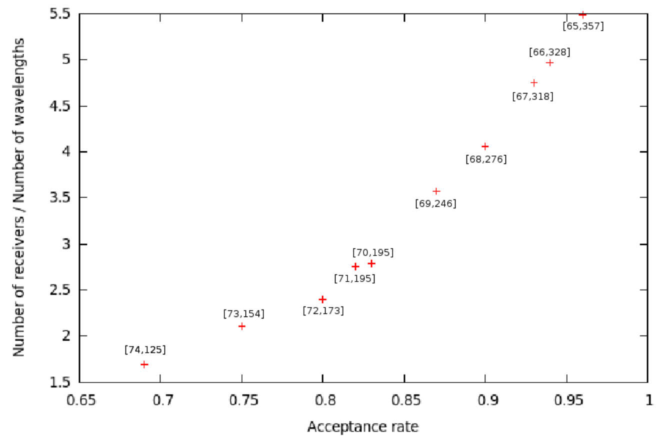

Fig. 1. Influence of the acceptance rate τ on $[z, W]$

We see that if τ increases then the ratio z/W increases too. If the acceptance rate is high then the pairs of elements tend to be rejected and packed later. The number of receivers thus increases whereas the number of wavelengths decreases. So, we know that when the load of traffic is high in the ring (i.e. the number of available wavelengths is small), solutions can be found by increasing the acceptance rate. Symmetrically if the number of wavelengths is large, then we can save receivers by decreasing the acceptance rate.

B. Influence of the pairing method

Figure 2 and 3 depict the influence of using or not the pairing method in step 2 of our heuristic. Figure 2 has been computed on an instance with ATA spatial distribution and normal (N(μ, 0.2μ)) distribution for the size of the connection whereas Figure 3 has been computed on an instance with RGR traffic distribution. On each we compare three curves. The straight line represents the minimal number of receivers (z_{min}) for a given traffic.

$$z_{min} = \sum_{i \in V} \left\lceil \frac{\sum_j T[j,i]}{C} \right\rceil$$

We notice that this lower bound, as it is not dependant on the number of available wavelengths, may not be close to the optimal solution when the number of available wavelengths is small. The two other curves represent the number of receivers required for a given traffic and a given number of wavelengths using both with and without the method of pairing.

We see with these curves, that the solution without pairing is not highly affected by the number of available wavelengths. On the other hand, the solution with pairing performs extremely well when the wavelength constraint is not tight but leads to worse results when the assignment of traffic becomes the bigger problem.

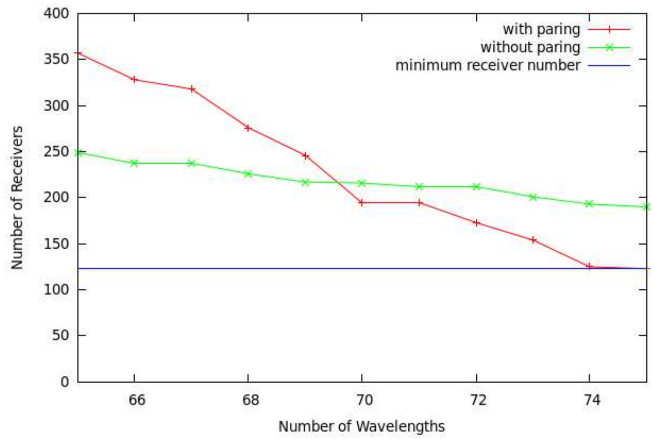

Fig. 2. Influence of the pairing method (ATA spatial distribution, Size of connections normally distributed)

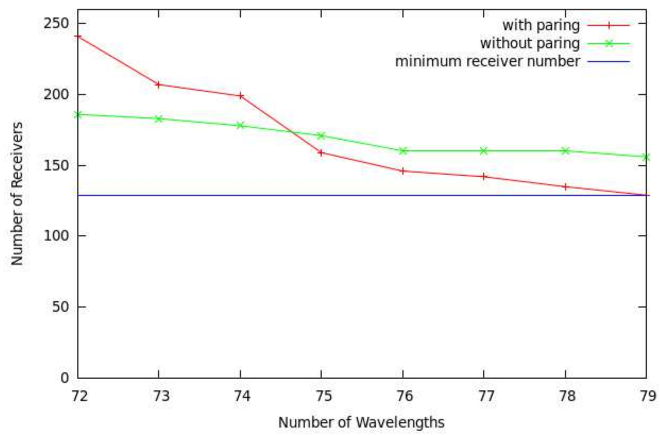

Fig. 3. Influence of the pairing method (RGR spatial traffic)

As we want to statut on the performance of the heuristic with pairing, we have to study a large number of instances. Nevertheless, for a given traffic matrix T, it is difficult to say if the wavelength constraint is either tight or open as the wavelength assignment problem is itself a difficult problem.

The MWLP heuristic, presented in [15], provides a solution that have the minimum number of receivers z_{min} and a number of wavelengths that we note W_{max}. For each MRP instance that has more than/exactly W_{max} available wavelengths we are able to find an optimal solution. Thus, our interest is in the MRP instances that have less than W_{max} available wavelengths.

Let W_{min} be the number of wavelengths used by a solution of a efficient wavelength assignment heuristic for the given matrix T. Such a solution can be provided, for instance, by the arc-colouring based heuristic presented in [14]. It seems reasonable to consider W_{min} as the minimal number of wavelengths required. In other word, we will consider the MRP instances which have more than W_{min} available wavelengths.

We generated 500 random traffic matrix for 16-node ring networks using RGR distribution. From a same matrix we generated three types of instances. In the first type, the number of available wavelengths is *hardly constraint* (i.e. $W \in (W_{min}, W_{min} + \frac{W_{max}-W_{min}}{3})$). In the second type, the number of available wavelength is *tight* (i.e. $W \in [W_{min} + \frac{W_{max}-W_{min}}{3}, W_{min} + \frac{2(W_{max}-W_{min})}{3}]$) and finally in the third one, the number of available wavelengths is *open* (i.e. $W \in (W_{min} + \frac{2(W_{max}-W_{min})}{3}, W_{max})$). In all three cases $C = 32$. The results of this experiment is showed in Figure 4. In this figure, $x\%$ means that the average solution has $x\%$ more receivers than the optimal solution for a number of available wavelength equals to W_{max} (W_{max} is provided by the MWLP heuristic).

	Open	Tight	Hard
With pairing	5.6%	20%	29.7%
Without pairing	14%	23.5%	27.5%

Fig. 4. Influence of the pairing method (Exhaustive simulations)

As expected the results presented in Figure 4 confirm the observations made on the instance of Figures 2 and 3 when the wavelength constraint is opened. In that case the heuristic with pairing performs well, increasing the minimum number of receivers by only 5.6 percent whereas the heuristic without paring get a 14 percent. When the wavelength constraint is tight the pairing method get also slightly better results than its opponent. Finally, the experience shows that the performance of the pairing method is comparable when the wavelength constraint is hard.

Conclusion and further work

The paper presents a part of our work dedicated to the dimensioning of all-optical ring networks with POADM node architecture. A POADM node can emit on all available wavelengths but it can only read on a subset of available wavelengths. The network infrastructure cost (CAPEX) of such a network can therefore be brought down by reducing the number of receivers present in the nodes. Our goal was to minimize this number when the number of available wavelengths is limited. We proved that the corresponding optimization problem is NP-complete. The heuristic we proposed is based upon a preliminary matching of pairs of grouped transmission which attempts to "wipe out" their shape irregularities. We proposed this coupling in order to obtain an efficient wavelength assignment. The exhaustive simulation results show the advantage of the introduction of pairing. They also exhibit that the heuristic results converge to the optimal solution when the number of available wavelengths is unlimited.

As a further work we consider studying together the problem we discussed here with the one which we treated before in [15], and which consisted of minimizing the wavelength number with the given, minimal number of receivers. We will want thus to formulate and solve a bi-criteria problem.

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